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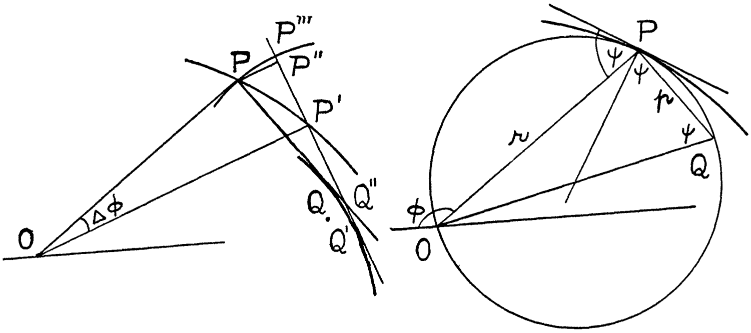
Whittemore may be obtained geometrically as follows: Consider two neighboring points, P and P', on the given curve and let O be the pole, Q and Q' the points of tangency of the perpendiculars with their envelope. Then the intersection Q'' of PQ and $P'Q'$ lies on the circle through O, P and P'. When P' approaches P as a limit the circle has for its limit a circle tangent to the given curve at P and passing through O. This gives a geometrical construction for Q. If ψ is the angle between OP and the tangent to the given curve then $\angle\, OQP = \psi$ and $\tan \psi = r/p$. But the well-known formula of polar coördnaites gives $\tan \psi = r/(dr/d\varphi)$ and hence $p = dr/d\varphi$. Consider that involute of the envelope (Q) which passes through P and let it cut $Q'P'$ in P'''; let P'' be the foot of the perpendicular from P upon $Q'P'$. Then $P'''Q' - PQ = \Delta s$ and $P'''Q' = P'''P'' + r \sin \Delta\varphi + p + \Delta p$ and hence



$$\frac{P'''P''}{\Delta\varphi} + \frac{r \sin \Delta\varphi}{\Delta\varphi} + \frac{\Delta p}{\Delta\varphi} = \frac{\Delta s}{\Delta\varphi}.$$

On taking the limit we have

$$r + \frac{dp}{d\varphi} = \rho,$$

since $P'''P''$ is equal to the length of the perpendicular from P to the tangent to the involute at P''', and hence it is an infinitesimal of the second order.

2892 [1921, 184]. Proposed by R. T. McGREGOR, Bangor, Calif.

Two parabolas have parallel axes. Prove that their common chord bisects their common tangent.

I. Solution by Marcia L. Latham, Hunter College.

Let the axes be rectangular, with the x-axis parallel to the axes of the parabolas; let P_1 and P_2, respectively, be the points of contact of the common tangent with the two parabolas, and P_3, the midpoint of P_1P_2.

Then the equations of the parabolas will be

$$y^2 + 2B_1x + 2C_1y + D_1 = 0, \tag{1}$$
$$y^2 + 2B_2x + 2C_2y + D_2 = 0. \tag{2}$$

The tangent to (1) at P_1 is

$$yy_1 + B_1(x + x_1) + C_1(y + y_1) + D_1 = 0.$$

But this passes through P_2; therefore,

$$y_1y_2 + B_1(x_1 + x_2) + C_1(y_1 + y_2) + D_1 = 0. \tag{3}$$

Again the tangent to (2) at P_2 passes through P_1; therefore,

$$y_1y_2 + B_2(x_1 + x_2) + C_2(y_1 + y_2) + D_2 = 0. \tag{4}$$

Subtracting (4) from (3), we can write

$$2(B_1 - B_2)\frac{x_1 + x_2}{2} + 2(C_1 - C_2)\frac{y_1 + y_2}{2} + (D_1 - D_2) = 0. \tag{5}$$

But the equation of the common chord is

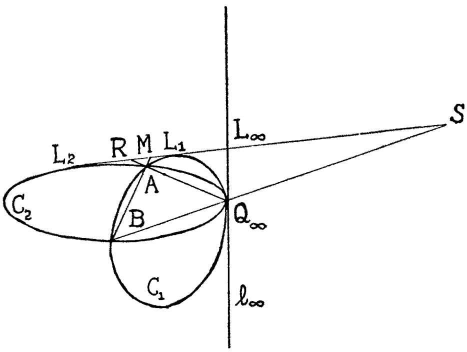
$$2(B_1 - B_2)x + 2(C_1 - C_2)y + (D_1 - D_2) = 0, \tag{6}$$

and thus (5) states that the coördinates of P_3 satisfy (6) and the common chord bisects the common tangent.[1]

II. Solution by W. E. Cleland, Princeton University.

Let the two conics C_1 and C_2 be tangent to the line at infinity of a Euclidean plane at Q_∞. Then C_1 and C_2 are parabolas whose axes are parallel. Let a common tangent be L_1L_2, tangent to C_1 at L_1 and to C_2 at L_2, and intersecting the line at infinity in L_∞. The common chord is AB, intersecting L_1L_2 in M. Let the intersections of AQ_∞ and BQ_∞ with L_1L_2 be R' and S, respectively.

Since the line L_1L_2 is tangent to C_1 at L_1, L_1 is a double point in the involution determined on L_1L_2 by the pairs of points $R\,S$, and $M\,L_\infty$. Similarly, L_2 is the other double point of the same involution. Hence M is the harmonic conjugate of L_∞ with respect to L_1 and L_2. M is therefore the midpoint of L_1L_2. (For proofs of these statements see Veblen and Young, *Projective Geometry.*)



Also solved by William Hoover, L. C. Mathewson, J. B. Reynolds, J. K. Whittemore, and F. L. Wilmer.

NOTES AND NEWS.

It is hoped that readers of the Monthly will coöperate in contributing to the general interest of this department by sending items to H. P. Manning, 69 Weymouth St., Providence, R. I.

Charles Albert Fischer, Seabury professor of mathematics at Trinity College, Hartford, Conn., died December 7,[2] 1922. He was born at Wheaton, Ill., October 3, 1884, and received the degree of A.B. from Wheaton (Ill.) College in 1905. He spent 1909–1912 in graduate study in mathematics and astronomy at the University of Illinois (A.M., 1910) and University of Chicago (Ph.D., 1912). His thesis, published in the *American Journal of Mathematics*, 1913, was entitled: "A generalization of Volterra's derivative of a function of a curve." At the meeting of the American Mathematical Society at the University of Rochester last September he gave, by invitation, an hour lecture on "Functions of lines." He was a charter member of the Association.

Arnold Sommerfeld, professor of mathematical physics at the University of Munich, will be in residence at the University of Wisconsin for the first semester of the academic year 1922–1923, holding the Karl Schurz Memorial Professorship in the University for that period. Professor Sommerfeld is expected to give a

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[1] The solution will have a particularly simple form if we take the common tangent and the line through its mid-point parallel to the axes of the parabolas for coördinate axes, giving us a system of oblique coördinates. The common chord will then appear as a line through the origin —Editors.

[2] The date is given incorrectly as December 9 in *Science*, December 15.